E-150 ASSIGNMENT 3 (100 POINTS)

MODELING AND SIMULATION OF ROBOTIC 3D PRINTERS

Due: Thursday, October 24th, 3:30 pm

In this project, you will simulate the operation of a free-form robotic 3D printer. Referring to Figure 1, the printer arm deposits electrically charged droplets above a substrate, with the substrate electrically charged in certain areas to attract the droplets. Using the simulation, you will employ a genetic algorithm to find control parameters that generate a printed pattern as close to a desired pattern as possible.

DYNAMICS OF 3D PRINTER ARM

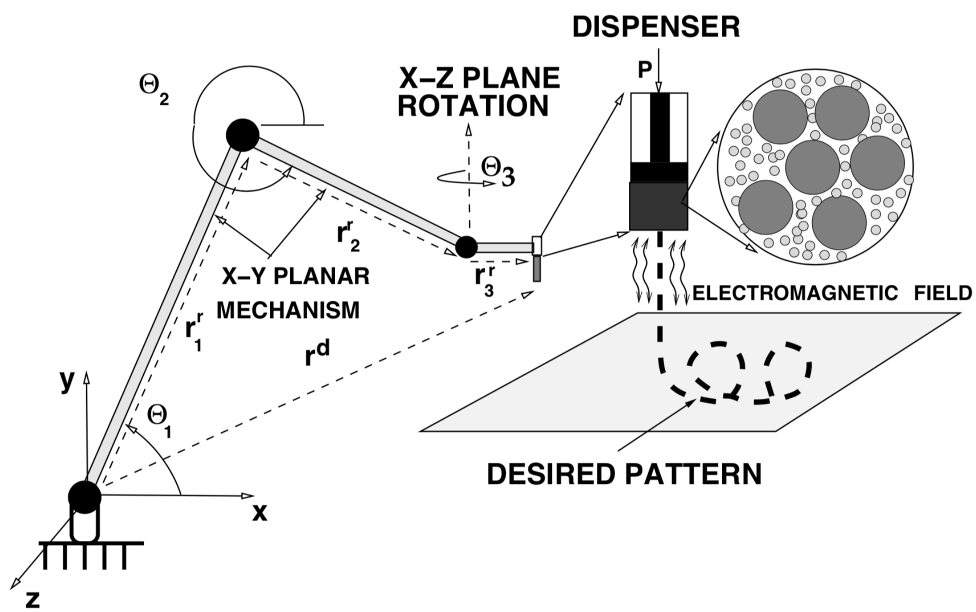

Figure 1: Coordinate System and Robot Schematic

We model the 3D printer arm as a three-rod linkage, with the first end of the first rod fixed and the free end of the third rod having the droplet dispenser. The first two rods are $x-y$ planar, and the third rod is $x-z$ planar, with all of their angular velocities constant. **Note that the coordinates are such that the y axis is vertical.** Given initial angular positions $\Theta_j^0,\ j = \{1, 2, 3\}$, angular velocities $\dot{\Theta}_j$, and rod lengths L_j, we can add up the rod rotations, as well as the position of the fixed end $\boldsymbol{r}_0$, to get the dispenser position components $\boldsymbol{r}^d = \boldsymbol{r}_0 + (x_d, y_d, z_d)$:

$$x_d = L_1 \cos\Theta_1 + L_2 \cos\Theta_2 + L_3 \sin\Theta_3 \tag{1}$$

$$y_d = L_1 \sin\Theta_1 + L_2 \sin\Theta_2 \tag{2}$$

$$z_d = L_3 \cos\Theta_3 \tag{3}$$

$$\Theta_j = \Theta_j(t) = \Theta_j^0 + \dot{\Theta}_j t \tag{4}$$

The time derivative of the dispenser position yields its velocity components $\boldsymbol{v}^d = (\dot{x}_d, \dot{y}_d, \dot{z}_d)$:

$$\dot{x}_d = -L_1\dot{\Theta}_1 \sin\Theta_1 - L_2\dot{\Theta}_2 \sin\Theta_2 + L_3\dot{\Theta}_3 \cos\Theta_3 \tag{5}$$

$$\dot{y}_d = L_1\dot{\Theta}_1 \cos\Theta_1 + L_2\dot{\Theta}_2 \cos\Theta_2 \tag{6}$$

$$\dot{z}_d = -L_3\dot{\Theta}_3 \sin\Theta_3 \tag{7}$$

To tie this in with the droplet dynamics, when the printer arm deposits a droplet, we initialize the droplet's position and velocity $\boldsymbol{r}_i^0, \boldsymbol{v}_i^0$ with the current $\boldsymbol{r}^d, \boldsymbol{v}^d$ at that time, adding a relative droplet dispenser velocity $\Delta\boldsymbol{v}^d$ for the added velocity of "pushing" the droplet out:

$$\boldsymbol{r}_i^0 = \boldsymbol{r}^d, \quad \boldsymbol{v}_i^0 = \boldsymbol{v}^d + \Delta\boldsymbol{v}^d \tag{8}$$

This gives us an initial condition for integrating to find a droplet's position and velocity.

DYNAMICS OF DROPLETS

We model the deposited droplets as lumped masses, moving as solid spheres of radius R. Using Newton's second law, accounting for gravitational, electric, and drag forces, the governing equation for some i-th droplet is:

$$m_i\ddot{\boldsymbol{r}}_i = \boldsymbol{\Psi}_i^{\text{tot}} = \boldsymbol{F}_i^{\text{grav}} + \boldsymbol{F}_i^{\text{elec}} + \boldsymbol{F}_i^{\text{drag}} \tag{9}$$

Where m_i is the mass of the droplet, $\ddot{\boldsymbol{r}}_i$ is the droplet's acceleration, and the three $\boldsymbol{F}$ are the gravitational, electric, and drag forces respectively.

In the coordinate system in Figure 1, the y axis is vertical, so we note the gravitational force is defined as

$$\boldsymbol{F}_i^{\text{grav}} = (g_x, g_y, g_z) = (0, -m_i g, 0) \tag{10}$$

where g is gravitational acceleration. For the electrical force, we model the droplet as a lumped charge with some charge q_i and the charged substrate as a set of N_c fixed-position point charges with charge q_p and position $\boldsymbol{r}_p$. Then, the electrical field and corresponding force is simply a sum of the electrical forces, neglecting the electrical forces between droplets:

$$\boldsymbol{F}_i^{\text{elec}} = \sum_{p=1}^{N_c} \frac{q_p q_i}{4\pi\epsilon\|\boldsymbol{r}_i - \boldsymbol{r}_p\|^2}(\boldsymbol{r}_i - \boldsymbol{r}_p) \tag{11}$$

Where ϵ is the permittivity, which we approximate as the free-space permittivity $\epsilon = \epsilon_0$ provided in the table at the end of this document. Lastly, the drag force depends on the geometry of the droplet and the properties of the surrounding medium:

$$\boldsymbol{F}_i^{\text{drag}} = \frac{1}{2}\rho_a C_{Di}\|\boldsymbol{v}^f - \boldsymbol{v}_i\|(\boldsymbol{v}^f - \boldsymbol{v}_i)A_i^D \tag{12}$$

Where ρ_a is the density of the surrounding medium, C_{Di} is the drag coefficient, $\boldsymbol{v}^f$ is the velocity of the surrounding medium, $\boldsymbol{v}_i = \dot{\boldsymbol{r}}_i$ is the droplet velocity, and $A_i^D = \pi R^2$ is the drag reference area. To determine the drag coefficient, we first determine the Reynolds number of the droplet Re:

$$Re = \frac{2R\rho_a \|\boldsymbol{v}^f - \boldsymbol{v}_i\|}{\mu_f} \tag{13}$$

Where μ_f is the viscosity of the surrounding medium. Then, the drag coefficient is a piecewise function of Re:

$$C_{Di} = \begin{cases} \frac{24}{Re}, & 0 < Re \leq 1 \\ \frac{24}{Re^{0.0646}}, & 1 < Re \leq 400 \\ 0.5, & 400 < Re \leq 3 \times 10^5 \\ 0.000366 Re^{0.4275}, & 3 \times 10^5 < Re \leq 2 \times 10^6 \\ 0.18, & Re > 2 \times 10^6 \end{cases} \tag{14}$$

Thus we have the governing equation for the droplet's motion, which we will numerically integrate using the Forward Euler method.

EFFECTIVE MATERIAL PROPERTIES OF DROPLETS

The droplets are assumed to be a mixture of two phases (materials), which has an effective density ρ^* and electrical charge capacity per unit volume q^*, assuming both phases may carry some kind of electrical charge per unit volume. These effective properties are needed for the lumped droplet charge $q_i = V_i q^*$, and mass $m_i = V_i \rho^*$.

The effective density formula and charge capacity is exact based on the volume fraction and the two phases' individual density and charge capacity $\rho_1, \rho_2, \langle q_1 \rangle, \langle q_2 \rangle$:

$$\rho^* = (1 - v_2)\rho_1 + v_2 \rho_2 \tag{15}$$

$$q^* = (1 - v_2)\langle q_1 \rangle + v_2 \langle q_2 \rangle \tag{16}$$

These equations can be used to determine the effective properties for the droplets. We note that these properties are equal for all deposited droplets, and only need to be calculated once.

FORWARD EULER INTEGRATION

We employ equations 9 (and the other equations needed to evaluate this one) and solve for $\ddot{\boldsymbol{r}}_i$. We use a Forward Euler scheme to integrate and find the position and velocity of some i-th droplet:

$$\boldsymbol{r}_i(t + \Delta t) = \boldsymbol{r}_i(t) + \Delta t \boldsymbol{v}_i(t) \tag{17}$$

$$\boldsymbol{v}_i(t + \Delta t) = \boldsymbol{v}_i(t) + \Delta t \ddot{\boldsymbol{r}}_i(t) \tag{18}$$

The robot arm dynamics dictate a droplet's initial conditions $\boldsymbol{r}_i(t = 0), \boldsymbol{v}_i(t = 0)$. Note that once a droplet reaches the substrate, the position should not change any more as we assume the droplet binds to the print bed.

GENETIC ALGORITHM

This assignment has two parts: (1) a model where you implement the full physics model of the droplet dynamics and numerically integrate to determine the final pattern and (2) a simplified model in which we neglect the effects of drag and "turn off" the electrical field, then use a genetic algorithm to find the angular velocities that best recreate a given pattern. The free design parameters are the robot arm angular velocities and the relative dispenser velocity. As such, we formulate the following design string:

$$\mathbf{\Lambda}^i = \{\dot{\Theta}_1, \dot{\Theta}_2, \dot{\Theta}_3, \Delta \boldsymbol{v}^d\} \tag{19}$$

A simple cost function in this case, if given a desired position $\boldsymbol{r}_i^{des}$ for each of N_d deposited droplets, would be:

$$\Pi = \frac{\sum_{i=1}^{N_d} \|\boldsymbol{r}_i^{des} - \boldsymbol{r}_i^{gen}\|}{\sum_{i=1}^{N_d-1} \|\boldsymbol{r}_i^{des} - \boldsymbol{r}_{i+1}^{des}\|} \tag{20}$$

where $\boldsymbol{r}_i^{gen} = \boldsymbol{r}_i^{gen}(\mathbf{\Lambda}^i)$ are the droplet positions generated by a given genetic string. Note that the denominator of this expression is the *arc length* of the desired pattern, and is used to nondimensionalize the cost function and ensure that the tolerance scales with the size of the pattern. We also assume that $\Delta \boldsymbol{v}^d$ is only in the vertical downward direction (a single component) so it only adds one value to the design string rather than three components. Initial ranges of values are given in the variable glossary.

PART 1: FULL MODEL WITH ELECTRICAL FORCES

In the first part of this assignment we model the droplets' dynamics, with droplets deposited for each of N_t time steps, not including t = 0, for a total droplet count $N_d = N_t$. We place several point charges on a print surface at $y = 0$, with a 10-by-10 grid of points each with charge q_p. Center the grid at the origin within a square with sides of length L_{bed}. To generate the grid positions (same for both x and z components), you can use `linspace(-`L_{bed}`/2, `L_{bed}`/2, 10)` and `meshgrid()`. We model the printer arm extruding for a total simulation time $T = 3$ seconds, such that the time step size $\Delta t = T/N_t$ with $\Delta t = 0.001$ seconds. After all the droplets are extruded, continue the simulation with the same time step size until all droplets land on the substrate.

1. *For all requested plots below, plot the point charges on the same figure. For visualization, let the boundary created by the point charges define the boundary created by the print bed.*

2. Provide a plot of the final droplet pattern in a "top-down" ($z - x$) view. Color each droplet according its flight time (i.e. time in the air) using `scatter` and show the color axis with `colorbar`. Be sure to label your color-bar with title and units.

3. Provide a 3D plot of the tool path of the robot arm's dispenser end with the beginning and end of the tool-path marked on the same plot as the print pattern on the print bed with the first and last droplet marked. Be sure to label these points using a legend.

4. Explain why you think the resulting droplet pattern occurred.

5. Set the electrical force to 0. Compare the animations and final configurations of the droplets with and without the electrical force. What role does the electrically charged substrate have on the motion of the droplets? It is up to you how to present this comparison, but be sure it is clear in your report. What adversary force does the electrical force assist against?

PART 2: SIMPLIFIED FREE FALL MODEL WITH GENETIC ALGORITHM

For the simplified model, we change the following:

- Remove drag from the model.

- Remove the electrical forces, leaving only gravity.

In doing so, our droplet dynamics reduces to a free fall model. The extruder model does not change from Part 1. Because our droplets only move by gravity, they are under constant acceleration, and thus the final position of the droplets can be solved for analytically. You should solve for that analytical solution and use it in your code. Do **not** use time stepping for this part of the project.

Here, we provide a data file `robotprint_data.mat` which contains a variable `ri`, a $3 \times N_d$ array where each column is a droplet's final position components with the columns ordered according to extrusion time. The aim of your genetic algorithm is to try and reproduce the design string $\mathbf{\Lambda}^{des}$ that produced the desired droplet positions $\boldsymbol{r}_i^{des}$ by minimizing Π. Set the tolerance of this cost function to $Tol = 2 \times 10^{-2}$.

1. Analytically solve for the final position of the droplets on the print if they are in free fall. Show the steps to your derivation. *Hint: Use constant acceleration kinematic equations from physics.*

2. Describe the parameters you used in your GA. Use $S = 100$ genetic strings, $P = 10$ parents which generate $P = 10$ children in the same ordering as previous projects.

3. Provide a $z - x$ "top-down" plot of the final droplet pattern on the same plot as the desired droplet pattern provided in the data file. How well does your design string follow the desired pattern? Use the provided helper function `comparepattern.p` to compare your generated pattern to your desired pattern.

4. Provide a convergence plot showing the total cost of the best design, the mean of all parent designs, and the mean of the overall population for each generation.

5. Report your best-performing 4 designs in a table similar to the following.

DESIGN	Λ_1	Λ_2	Λ_3	*etc*	Π
1					
2					
3					
4					

Table 1: The top 4 system parameter performers.

Note, `comparepattern.p` works by running the simulation for your inputted design string, calculating $\Pi(\mathbf{\Lambda}^i)$ and outputting the comparison plot. As such, its access is restricted and you cannot view the content of the file. You may **not** use this function to run the simulation for your GA code. Credit will only be received for codes which create the model themselves. This function is simply a tool to allow you to easily visualize your results and generate plots for your report. The function has the following syntax: `PI = comparepattern(t1d,t2d,t3d,vdR)` where `t1d`= $\dot{\Theta}_1$, `t2d`= $\dot{\Theta}_2$, `t3d`= $\dot{\Theta}_3$, `vdR`= $\Delta\boldsymbol{v}^d$ and are all 1×1 scalars representing the parameters from a single genetic string $\mathbf{\Lambda}^i$.

For your report, please follow the format provided on bCourses in the file *E150ReportFormat.pdf*. Integrate the questions asked above into your report where you deem appropriate.

To submit your assignment:

1. Upload a zip file to bCourses containing your code (as a .m file, .py file, etc) and any supporting files required to run your code.
2. Upload a PDF version of your report separately from the zip file (this simplifies grading). To save paper, **electronic-only submission of your report is encouraged**. This makes it possible to include colored graphs without paying for color printing. If you prefer to receive feedback on a printed copy, you may still submit one to the box outside 6102 Etcheverry Hall or during lecture.

VARIABLE GLOSSARY

Symbol	Type	Units	Value	Description
$\boldsymbol{r}_0$	3×1 vector	m	(0, 0.5, 0)	Fixed arm end position
x_d, y_d, z_d	scalars	m	Equations 1-3	Dispenser position
Θ_j	scalars	radians	Equation 4	Rod angles
$\Theta_1^0, \Theta_2^0, \Theta_2^0$	scalars	radians	$\pi/2, 0, 0$	Initial rod angles
$\dot{\Theta}_1$,$\dot{\Theta}_2$,$\dot{\Theta}_3$	scalar	rad/s	0.2,-0.2,10	Link 1,2,3 angular velocity
L_{bed}	scalar	m	.8	charged grid side length
L_1, L_2, L_3	scalars	m	.3, .2, .08	Rod lengths
$\dot{x}_d, \dot{y}_d, \dot{z}_d$	scalars	m/s	Equations 5-7	Dispenser velocity
$\boldsymbol{r}_i^0$	3×1 vector	m	Equation 8	Initial droplet position
$\boldsymbol{r}_i$	3×1 vector	m	Equation 27	Droplet position
$\boldsymbol{v}_i^0$	3×1 vector	m/s	Equation 8	Initial droplet velocity
$\boldsymbol{v}_i$	3×1 vector	m/s	Equation 28	Droplet velocity
$\ddot{\boldsymbol{r}}_i$	3×1 vector	m/s^2	Equation 9	Droplet acceleration
$\boldsymbol{F}_i^{\text{grav}}, \boldsymbol{F}_i^{\text{elec}}, \boldsymbol{F}_i^{\text{drag}}$	3×1 vectors	N	Equations 10-12	Gravitational, electric, and drag forces
g	scalar	m/s^2	9.81	Gravitational acceleration
m_i	scalar	kg	$V_i\rho^*$	Droplet mass
ϵ	scalar	F/m	8.854×10^{-12}	Electric permittivity
q_i	scalar	C	$V_i q^*$	Droplet charge
$\boldsymbol{r}_p$	3×1 vector	m	(varies)	Point charge position(s)
V_i	scalar	m^3	$(4/3)\pi R^3$	Droplet volume
R	scalar	m	0.001	Droplet radius
v_2	scalar	unitless	0.25	Phase 2 volume fraction
ρ^*	scalar	kg/m^3	Equation 24	Effective density
ρ_1	scalar	kg/m^3	2000	Phase 1 density
ρ_2	scalar	kg/m^3	7000	Phase 2 density
q^*	scalar	C / m^3	Equation 26	Effective charge capacity
q_1	scalar	C / m^3	0	Phase 1 charge capacity
q_2	scalar	C / m^3	+1e-3	Phase 2 charge capacity
$\Delta\boldsymbol{v}^d$	3×1 vector	m/s	(0, -1.2, 0)	Relative extrusion velocity
q_p	scalar	C	-8×10^{-5}	Grid pixel charge
$\boldsymbol{v}^f$	3×1 vector	m/s	(0.5, 0, 0.5)	Surrounding medium velocity
ρ_a	scalar	kg/m^3	1.225	Surrounding medium density
μ_f	scalar	Pa/s	1.8×10^{-5}	Surrounding medium viscosity
C_{Di}	scalar	unitless	Equation 14	Drag coefficient
A_i^D	scalar	m^2	πR^2	Drag reference area
Δt	scalar	seconds	0.001	Time Step Size
T	scalar	seconds	3	Total Simulation Time
$\dot{\Theta}_1^-, \dot{\Theta}_1^+$	scalar	rad/s	[15, 16]	Search bounds for $\dot{\Theta}_1$
$\dot{\Theta}_2^-, \dot{\Theta}_2^+$	scalar	rad/s	[15, 16]	Search bounds for $\dot{\Theta}_2$
$\dot{\Theta}_3^-, \dot{\Theta}_3^+$	scalar	rad/s	[6, 7]	Search bounds for $\dot{\Theta}_3$
$\Delta v^{d,-}, \Delta v^{d,+}$	scalar	m/s	[-3.5, -3]	Search bounds for Δv^d