ME 104: ENGINEERING MECHANICS II
Department of Mechanical Engineering
University of California at Berkeley
December 14, 2017

Spring Semester 2018

Professor Oliver M. O'Reilly

Assignments for Homework No. 7
Due by 1:00pm on Friday March 9, 2018

Class Announcements

(i) The first four problems in this week's homework involve angular momentum $\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$ and the angular momentum theorem $\dot{\mathbf{H}}_O = \mathbf{r} \times \mathbf{F}$.

(ii) The second set of problems are based on impact of a particle(s). To solve these problems, I recommend that you use the formulae discussed on pages 98–103 of the primer. These formulae relate the pre– and post–impact velocity vectors:

$$\begin{aligned} \mathbf{v}_1' &= (\mathbf{v}_1 \cdot \mathbf{t}_1)\mathbf{t}_1 + (\mathbf{v}_1 \cdot \mathbf{t}_2)\mathbf{t}_2 + \frac{1}{m_1 + m_2}\left((m_1 - em_2)\mathbf{v}_1 \cdot \mathbf{n} + (1+e)m_2\mathbf{v}_2 \cdot \mathbf{n}\right)\mathbf{n}, \\ \mathbf{v}_2' &= (\mathbf{v}_2 \cdot \mathbf{t}_1)\mathbf{t}_1 + (\mathbf{v}_2 \cdot \mathbf{t}_2)\mathbf{t}_2 + \frac{1}{m_1 + m_2}\left((m_2 - em_1)\mathbf{v}_2 \cdot \mathbf{n} + (1+e)m_1\mathbf{v}_1 \cdot \mathbf{n}\right)\mathbf{n}. \quad (1) \end{aligned}$$

These formulae state that the velocities in the tangential directions are unaffected by the impact, while those in the normal direction are substantially affected. In most impact problems the first thing to do is to determine expressions for $\mathbf{n}$, $\mathbf{t}_1$ and $\mathbf{t}_2$.

Homework Policy

Solutions to the written homeworks should be submitted to the drop-off box located on the 3rd floor of Etcheverry Hall by 1:00pm. The homework will be picked up at 1:00pm and late homeworks will *not* be graded. You are also expected to complete the electronic homework problems by 1:00pm on the same day that homework is due. We expect to return the corrected homeworks to you promptly and the solutions will be posted on **bcourses** by 3:00pm on the days the homework is due.

The E-problems can be accessed from your account at

WileyPlus

Here are some key points to take into consideration when submitting your homework solutions:

1. While I expect you to work with other students in the class on the homework, the final submitted solution should be yours alone. If we find violations of this policy, then the homeworks in question will be returned ungraded, and the possibility of not grading the student(s) homework for the remainder of the semester will be considered.
2. Where appropriate, your MATLAB/MATHEMATICA code should be submitted with your homework solutions. The code should be commented and contain your name and SID.
3. Points total for each homework problem are given below. Most of the points will be assigned for the method you use to solve the problem and not merely on getting the correct numerical answer.
4. No credit will be given for untidy and/or illegible homework solutions.
5. All vectors should be underlined.

Homework 7
8 Questions
(50 + 70 Points)

1. Problem 3/217 *(E-problem 10 Points)*

This should be a straightforward problem using the definitions of T, $\mathbf{H}_O$ and $\mathbf{G}$.

2. Problem 3/231 *(E-problem 10 Points)*

This is a very interesting problem: $\mathbf{H}_O$ of the system is conserved during the collision but $\mathbf{G}$ of the system is not.

3. Problem 3/246 *(E-problem 10 Points)*

This problem involves relating the rebound height to the coefficient of restitution. A closely related problem can be found on pages 104–106 of the Primer.

4. Problem 3/248 *(E-problem 20 Points)*

There are several approaches to this problem. One is to use the definition of e and conservation of linear momentum during the collision. Equivalently, one can simply apply (1).

5. Problem 3/216 *(10 Points)*

This should be a straightforward problem using the definition of $\mathbf{H}_O$ and the angular momentum theorem.

6. Problem 3/230 *(20 Points)* Four Steps Needed Here

You need to use the four steps here. With this you can use $\mathbf{F} = m\mathbf{a}$ to find an expression for the position vector of the particle as a function of time: $\mathbf{r}(t) = ut\mathbf{E}_x - \frac{1}{2}gt^2\mathbf{E}_y$. The rest of the problem involves calculating $\mathbf{H}_O$.

In this problem, you should notice that $\mathbf{G} \cdot \mathbf{E}_x$ is conserved, and $\mathbf{H}_O \cdot \mathbf{E}_z$ is not conserved.

7. Problem 3/237 *(20 Points)* Four Steps Needed Here Discussion Session Problem

To avoid confusion let's label r in the figure R and the angle θ requested in the solution as β. This is a very interesting problem - total energy is conserved and $\mathbf{H}_O \cdot \mathbf{E}_z$ is conserved. You need to prove these two statements (in Step IV) for full credit.

In Step I, use the representation $\mathbf{r} = r\mathbf{e}_r + z\mathbf{E}_z$. Because the particle moves on the surface $z = f(r)$, z is not independent of r but for the question that's being asked you don't need to deal with the function $f(r)$, so you simply treat r and z as independent variables. In the initial position, z is given and $\mathbf{v} = v_0\mathbf{e}_\theta$. In the final position, $z = 0$ and $\mathbf{v} = r_f\dot{\theta}_f\mathbf{e}_\theta + \dot{z}_f\mathbf{E}_z$. The key to the problem is to use the aforementioned conservations to solve for $\dot{\theta}_f$ and $\dot{z}_f$. Note that $\dot{z}_f < 0$.

Step III in this problem doesn't yield any information that's useful for solving the posed question. You should note however the similarities between this problem and 3/234 and 3/238.

8. Problem 3/260 *(20 Points)* Discussion Session Problem

A closely related, indeed very closely related problem, is discussed on Pages 106–107 of the Primer.

In this problem, you first need to determine the vectors $\mathbf{n}, \mathbf{t}_1$, and $\mathbf{t}_2$. One choice is

$$\mathbf{n} = \cos(30^\circ)\mathbf{E}_x - \sin(30^\circ)\mathbf{E}_y, \quad \mathbf{t}_1 = \cos(60^\circ)\mathbf{E}_x + \sin(60^\circ)\mathbf{E}_y, \quad \mathbf{t}_2 = \mathbf{E}_z. \tag{2}$$

I highly recommend not evaluating the sin and cosine functions in these expressions until the very end of the problem. You will also find the identities $\sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha) = \sin(\alpha \pm \beta)$ and $\cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) = \cos(\alpha \pm \beta)$ to be helpful.

Using the equations (1) for $\mathbf{v}_1'$ and $\mathbf{v}_2'$ in terms of $\mathbf{v}_1$ and $\mathbf{v}_2$, you will find, in order for $\mathbf{v}_1' = -v_1'\mathbf{E}_y$, an equation for θ:

$$\cos(\theta - 30^\circ) = \frac{3-e}{1+e}\cos(60^\circ). \tag{3}$$

The solutions θ to this equation are 82.33° and -22.33°. For full credit, illustrate the two impact scenarios corresponding to the two angles.

ME104: Engineering Mechanics II
Spring Semester 2018

Homework Solutions for Homework 7

Prepared by Oliver M. O'Reilly
(oreilly@berkeley.edu)

Problem 3/216

M, K & B 8th Edition

Given: $\underline{F} = F\underline{E}_y$

$\underline{r} = a\underline{E}_x + b\underline{E}_y + c\underline{E}_z$

$m = m$

$\underline{v} = v\underline{E}_x$

Determine $\underline{H}_o$ and $\dot{\underline{H}}_o$

Solution:

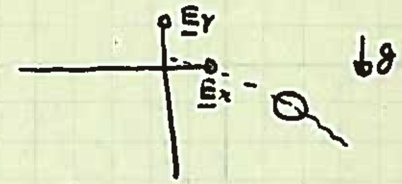
$$\underline{H}_o = \underline{r} \times m\underline{v} = -mbv\underline{E}_z + mvc\underline{E}_x$$

$$\dot{\underline{H}}_o = \underline{r} \times \underline{F} = aF\underline{E}_z - Fc\underline{E}_y$$

Note: can't determine $\dot{\underline{H}}_o$ by differentiating $mvc\underline{E}_x - mbv\underline{E}_z$ because you aren't told directly what $\underline{a}$ is. From $\underline{F} = m\underline{a}$ you can infer that

$$\underline{a} = +\frac{F}{m}\underline{E}_y$$

Problem 3/230



Given: $\underset{\sim}{r}(0) = \underset{\sim}{0}$, $\quad \underset{\sim}{v}(0) = u\underset{\sim}{E}_x$

Determine: $\underset{\sim}{H}_0$

Solution:

I Kinematics: $\quad \underset{\sim}{r} = x\underset{\sim}{E}_x + y\underset{\sim}{E}_Y + z\underset{\sim}{E}_z$

$$\Rightarrow \quad \underset{\sim}{a} = \ddot{x}\underset{\sim}{E}_x + \ddot{y}\underset{\sim}{E}_Y + \ddot{z}\underset{\sim}{E}_z$$

II Freebody

$-mg\underset{\sim}{E}_Y$

III $\underset{\sim}{F} = m\underset{\sim}{a}$:

$$m\ddot{x} = 0 \quad \Rightarrow \quad x(t) = x(t_0) + \dot{x}(t_0)(t-t_0)$$

$$m\ddot{y} = -mg \quad \Rightarrow \quad y(t) = y(t_0) + \dot{y}(t_0)(t-t_0) - \tfrac{1}{2}g(t-t_0)^2$$

$$m\ddot{z} = 0 \quad \Rightarrow \quad z(t) = z(t_0) + \dot{z}(t_0)(t-t_0).$$

IV Analysis: $\quad t_0 = 0, \quad \underset{\sim}{r}(0) = \underset{\sim}{0}, \quad \underset{\sim}{v}(0) = u\underset{\sim}{E}_x$

$$\Rightarrow \quad \underset{\sim}{r}(t) = ut\underset{\sim}{E}_x - \tfrac{1}{2}gt^2\underset{\sim}{E}_Y$$

$$\Rightarrow \quad \underset{\sim}{v}(t) = u\underset{\sim}{E}_x - gt\underset{\sim}{E}_Y$$

$$\Rightarrow \quad \underset{\sim}{H}_0 = \underset{\sim}{r} \times m\underset{\sim}{v} = m\left(ut\underset{\sim}{E}_x - \tfrac{1}{2}gt^2\underset{\sim}{E}_Y\right) \times \left(u\underset{\sim}{E}_x - gt\underset{\sim}{E}_Y\right)$$

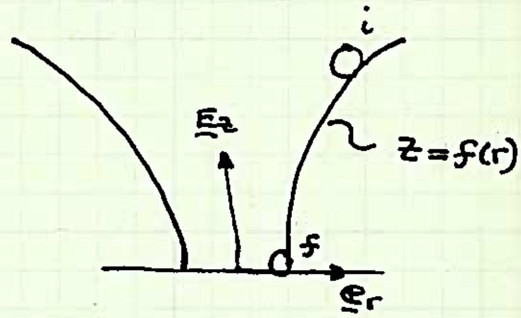
$$= -mgut^2\underset{\sim}{E}_z + \tfrac{m}{2}gt^2u\underset{\sim}{E}_z$$

$$= -\frac{mg}{2}ut^2\underset{\sim}{E}_z$$

Problem 3|2~~3~~4

Given $\underset{\sim}{v}_0 = v_0 \underset{\sim}{e}_\theta$ when $\underset{\sim}{r} = \underset{\sim}{r}_0$

Determine β where ~~Compute~~ β is the angle made by $\underset{\sim}{v}$ with $\underset{\sim}{e}_\theta$ when $r = .15R$ and $z = 0$



Solution:

I

$$\underset{\sim}{r} = r\underset{\sim}{e}_r + z\underset{\sim}{E}_z$$ where z should be considered as a function of r

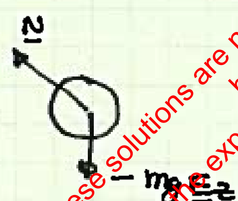
$$\underset{\sim}{v} = \dot{r}\underset{\sim}{e}_r + r\dot{\theta}\underset{\sim}{e}_\theta + \dot{z}\underset{\sim}{E}_z$$

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2)$$

$$\underset{\sim}{H}_0 \cdot \underset{\sim}{E}_z = mr^2\dot{\theta}$$

$$U = +mgz$$

II



Note that $\underset{\sim}{N} \cdot \underset{\sim}{e}_\theta = 0$

III

$\underset{\sim}{F} = m\underset{\sim}{a}$ doesn't give us any information that can be directly used to solve problem

IV

E is conserved: To see this $\dot{E} = \underset{\sim}{F}_{nc} \cdot \underset{\sim}{v} = \underset{\sim}{N} \cdot \underset{\sim}{v} = 0$

$\underset{\sim}{H}_0 \cdot \underset{\sim}{E}_z$ is conserved:

$$\dot{\underset{\sim}{H}}_0 \cdot \underset{\sim}{E}_z = (\underset{\sim}{r} \times \underset{\sim}{F}) \cdot \underset{\sim}{E}_z = (\underset{\sim}{E}_z \times \underset{\sim}{r}) \cdot \underset{\sim}{F}$$
$$= r\underset{\sim}{e}_\theta \cdot \underset{\sim}{F}$$
$$= 0$$

TOPS 35500

Now initially

$$z = R\cos 30^\circ$$

$$r = -R\sin 30^\circ + R + .15R$$

$$\dot{z} = 0 \qquad \dot{r} = 0 \qquad \underline{v}_0 = v_0\,\underline{e}_\theta$$

$$E = \frac{1}{2} m v_0^2 + mgR\cos 30^\circ$$

$$\underline{H}_0 \cdot \underline{E}_z = m r_0 v_0 = m v_0 (1.15 - \sin 30) R$$

Finally

$$z = 0$$

$$r = .15R \qquad \dot{r} = 0 \qquad \underline{v} = \dot{z}\,\underline{E}_z + r\dot{\theta}\,\underline{e}_\theta$$

$$\underline{H}_0 = m r_0^2 \dot{\theta} = (.15)^2 R^2 m \dot{\theta}_f$$

$$E = \frac{1}{2} m \left(\dot{z}_f^2 + (.15R)^2 \dot{\theta}_f^2 \right)$$

Solving :

$\underline{H}_0 \cdot \underline{E}_z$ is conserved $\Rightarrow$

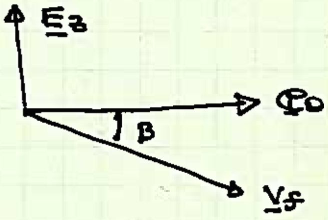
$$m R v_0 (1.15 - \sin 30) = m R^2 (.15)^2 \dot{\theta}_f$$

$$\dot{\theta}_f = \frac{v_0}{R} \frac{1.15 - \sin 30}{(.15)^2}$$

E is conserved :

$$\dot{z}_f^2 = v_0^2 + 2gR\cos 30^\circ - \frac{v_0^2}{\ } \frac{(1.15 - \sin 30)^2}{(.15)^2}$$

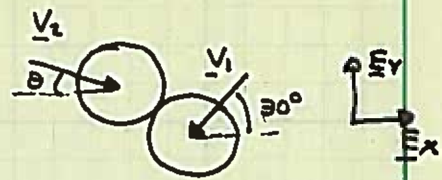
with given numbers: $R = 9\,m$ $\quad v_0 = 0.55\,m/sec$



$$\underline{v}_f = 2.38\,\underline{e}_\theta - 3.148\,\underline{E}_z \quad (m/sec)$$

$$\beta = 52.87^\circ$$

Problem 3/260



Given:

$$\underset{\sim}{V}_1 = V_1(-\cos 30\, \underset{\sim}{E}_X - \sin 30\, \underset{\sim}{E}_Y)$$

$$\underset{\sim}{V}_2 = V_2(\cos\theta\, \underset{\sim}{E}_X - \sin\theta\, \underset{\sim}{E}_Y), \quad V_2 = V_1$$

$$m_1 = m_2$$

$$\varepsilon = 0.8$$

Determine:

θ such that $\underset{\sim}{V}_1' = -V_1' \underset{\sim}{E}_Y$

Solution:

we first need to determine $\{\underset{\sim}{n}, \underset{\sim}{t}_1, \underset{\sim}{t}_2\}$

$$\underset{\sim}{n} = \cos 30\, \underset{\sim}{E}_X - \sin 30\, \underset{\sim}{E}_Y$$

$$\underset{\sim}{t}_1 = \cos 60\, \underset{\sim}{E}_X + \sin 60\, \underset{\sim}{E}_Y$$

$$\underset{\sim}{t}_2 = \underset{\sim}{E}_Z$$

Next: Linear momenta of both particles in $\underset{\sim}{t}_1$ & $\underset{\sim}{t}_2$ directions is conserved

$$\underset{\sim}{V}_1 \cdot \underset{\sim}{t}_1 = \underset{\sim}{V}_1' \cdot \underset{\sim}{t}_1 = V_1(-\cos 30 \cos 60 - \sin 30 \sin 60) = -V_1 \cos 30$$

$$\underset{\sim}{V}_1 \cdot \underset{\sim}{t}_2 = \underset{\sim}{V}_1' \cdot \underset{\sim}{t}_2 = 0$$

$$\underset{\sim}{V}_2 \cdot \underset{\sim}{t}_1 = \underset{\sim}{V}_2' \cdot \underset{\sim}{t}_1 = V_1(\cos\theta \cos 60 - \sin\theta \sin 60) = V_1 \cos(60 + \theta)$$

$$\underset{\sim}{V}_2 \cdot \underset{\sim}{t}_2 = \underset{\sim}{V}_2' \cdot \underset{\sim}{t}_2 = 0$$

Secondly: Linear momentum of the system in the $\underset{\sim}{n}$ direction is conserved.

$$\underset{\sim}{V}_1' \cdot \underset{\sim}{n} + \underset{\sim}{V}_2' \cdot \underset{\sim}{n} = \underset{\sim}{V}_1 \cdot \underset{\sim}{n} + \underset{\sim}{V}_2 \cdot \underset{\sim}{n} = -V_1 \cos 60 + V_1(\cos(\theta + 30))$$

Finally: Coefficient of Restitution

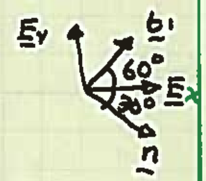
$$\underset{\sim}{V}_2' \cdot \underset{\sim}{n} - \underset{\sim}{V}_1' \cdot \underset{\sim}{n} = e(\underset{\sim}{V}_1 \cdot \underset{\sim}{n} - \underset{\sim}{V}_2 \cdot \underset{\sim}{n})$$

$$= -e V_1(\cos 60 + \cos(\theta - 30))$$

Solving the last 2 equations for $\underset{\sim}{v}_1' \cdot \underset{\sim}{n}$ and $\underset{\sim}{v}_2' \cdot \underset{\sim}{n}$

$$\underset{\sim}{v}_1' \cdot \underset{\sim}{n} = \frac{v_1}{2}\left[(1+e)\cos(\theta - 30) - (1-e)\cos 60 \right]$$

$$\underset{\sim}{v}_2' \cdot \underset{\sim}{n} = \frac{v_1}{2}\left[(1-e)\cos(\theta - 30) - (1+e)\cos 60 \right]$$



In summary

$$\underset{\sim}{v}_1' = v_1\left[-\cos 30\, \underset{\sim}{t}_1 + \frac{1}{2}\left[(1+e)\cos(\theta-30) - (1-e)\cos 60 \right] \underset{\sim}{n} \right]$$

$$\underset{\sim}{v}_2' = v_1\left[\cos(60+\theta)\, \underset{\sim}{t}_1 + \frac{1}{2}\left[(1-e)\cos(\theta-30) - (1+e)\cos 60 \right] \underset{\sim}{n} \right]$$

Now we want $\underset{\sim}{v}_1' = -v_1' \underset{\sim}{E}_y \Rightarrow \underset{\sim}{v}_1' \cdot \underset{\sim}{E}_x = 0$

$$\Rightarrow -\cos 30\, \underset{\sim}{t}_1 \cdot \underset{\sim}{E}_x + \frac{1}{2}\left[(1+e)\cos(\theta-30) - (1-e)\cos 60 \right] \underset{\sim}{n} \cdot \underset{\sim}{E}_x = 0$$

$$\Rightarrow \boxed{\cos(\theta - 30) = \frac{3-e}{1+e}\cos 60}$$

Given e we can solve this equation to determine θ. ++

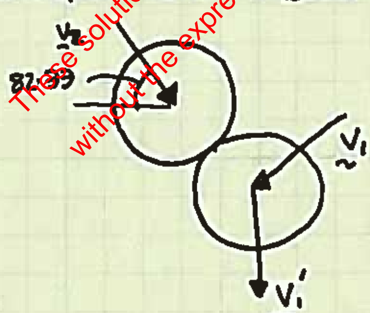
Here $e = 0.8$, $\cos(60) = 1/2$

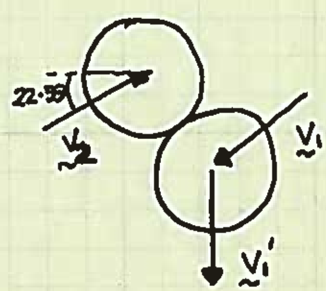
$$\Rightarrow \cos(\theta - 30) = \frac{2.2}{2(1.8)} = \frac{11}{18}$$

$$\Rightarrow \theta - 30 = \cos^{-1}\left(\tfrac{11}{18}\right) \quad \text{or} \quad 30 - \theta = \cos^{-1}\left(\tfrac{11}{18}\right)$$

$$\Rightarrow \theta = 82.33 \quad \text{or} \quad -22.33$$

These values of θ correspond to two different impact scenarios.





++ As $\cos(\theta - 30) = \dfrac{3-e}{2+2e} \leq 1$ $(e \in [0,1])$ we can't find θ for any e between 0 and 1.
For instance if the impact were purely plastic $(e = 0)$, then the problem wouldn't have a soln.